Skin-Friction Measurements in Laminar and Turbulent Flows Using Heated Thin-Film Gages

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Nomenclature

a = temperature coefficient of resistance of thin film
i = gage heating current
k = thermal conductivity
L = effective length of heated element
Nu = Nusselt number ($q_\omega L/k\Delta T$)
q = heat flux
R = resistance of gage
ΔT = temperature difference between thin-film heated element and local ambient (overheat)
u = velocity
w = width of heated element
y = distance normal to wall
η = yu_τ/v_ω
μ = viscosity
v = kinematic viscosity
ρ = density
σ = Prandtl number
τ = shear stress
ϕ = u/u_τ

Subscripts
e = effective
M = measured
ω = wall

Theme

THE objective of this research is an analysis of the calibration behavior of thin-film heated element gages in both laminar and turbulent flows. Attention is paid to the possible effects due to wall temperature distribution changes, which are discounted, and the effect of turbulence on viscosity, conductivity, and the average velocity profile in the wall region. The theory leads to a correction that can be applied to turbulent data to reduce them to the same calibration as that obtained in laminar flow. A criterion is developed for agreement, within limits, of the calibrations from the two flow regimes. From this it is possible to predict that the effects of turbulence are negligible in water flows but noticeable in air flows for specific gages.

Contents

The measurement of skin friction in air using heated thin-film gages has been described by Bellhouse and Schultz.[1] Brown[2] has derived a criterion for equivalence of calibration (in laminar and turbulent flows) based on confining heat transfer to the so-called laminar or linear sublayer of a turbulent flow. Owen[3] has derived a more restrictive criterion for agreement of calibrations to within 10%. This allows for variation of fluid viscosity within the sublayer, and a consequent difference in calibrations even when the thermal boundary layer of the gage is contained within this sublayer. Experimental results are presented here that resolve the problem of calibration in the two regimes. These experimental calibrations were performed by setting a variety of thin-film gages flush with the wall of a flat plate, which was placed symmetrically in the working section of a low-speed wind tunnel. Further calibrations were obtained on a flat plate towed in a water tank.

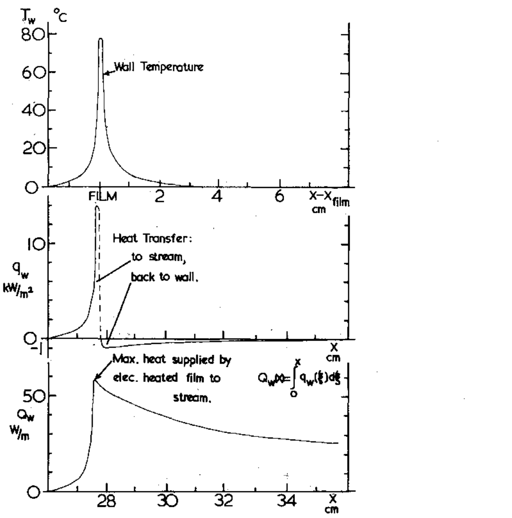

Fig. 1 Local heat transfer calculated from a measured wall temperature distribution.

The analysis of Bellhouse and Schultz,[1] and to some extent that of Brown,[2] depends on an assumed wall temperature profile that is a "top-hat" composed of two step functions. Figure 1 shows that this is not necessarily justified, although it

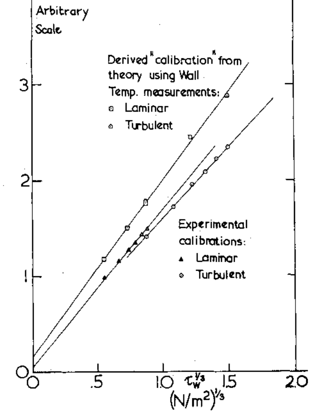

Fig. 2 Calibration of a heated element derived from surface temperature measurements, and compared with experimental calibration.

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Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Boundary Layers and Convective Heat Transfer—Turbulent.

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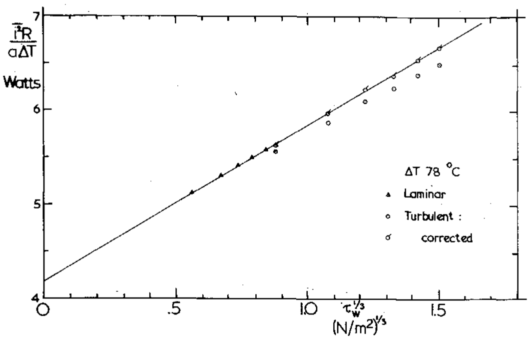

Fig. 3 Correction applied to thin-film calibration of Fig. 2.

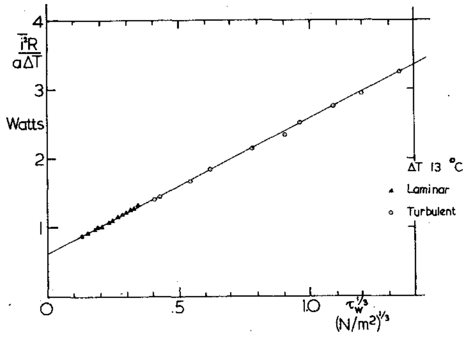

Fig. 4 Water calibration of thin-film gage.

affords a great theoretical convenience in the derivation of the calibration equation. Using measured wall temperature distributions, such as that shown in Fig. 1, it is possible to calculate the heat transfer to the boundary layer for several cases over the range of laminar and turbulent skin friction. Figure 1 illustrates one such calculation based on Lighthill's formula[4] for heat transfer from a flat plate. Figure 2 shows a "calibration" deduced in this way from wall temperature measurements. From this it can be seen that despite measurable changes in wall temperature profile these have little effect on the "calibration." The reasons for the observable differences in laminar and turbulent calibrations (see Fig. 2) must therefore be sought in the different nature of the two boundary layers themselves.

Using effective values of k and μ in a turbulent boundary layer that are greater than the laminar or true "wall" values, certain modifications are introduced into the boundary-layer approximations of the Navier-Stokes equations. Using linear variations, such as $\mu_e = \mu_\omega(1+\alpha y)$ and $k_e = k_\omega(1+\beta y)$, the thermal boundary-layer equation gives

$$Nu = 0.8072\left[\frac{\rho c_p \tau_\omega L^2}{\mu_\omega k_\omega}\right]^{1/3} + 0.10(2\beta-\alpha)L - 0.02869 \times \left[\frac{\mu_\omega k_\omega}{\rho c_p \tau_\omega L^2}\right]^{1/3}((2\beta-\alpha)L)^2 \equiv 0.8072\left[\frac{\rho c_p \tau_M L^2}{\mu_\omega k_\omega}\right]^{1/3} \quad (1)$$

where τ_M is the "measured" or interpreted value of local skin friction deduced from heat-transfer measurements.

From a thorough study of existing traverse data in the sublayer, including much obtained by the latest flow visualisation "particle" and "tracer" techniques, a quadratic fit is proposed for the average velocity profile in the region $\eta < 20$. This fit is $\phi = \eta(1-0.0208\eta)$ and can be used to deduce a value for $\alpha = 0.0416u_\tau/\nu$. If it is assumed that the variation of thermal conductivity in the sublayer ($\eta < 5$) is such that $2\beta < \alpha$, then Eq. (1) indicates that the turbulent calibration slope will be less than the laminar one. This is found to be true experimentally (Figs. 2 and 3). If it is further assumed that $\beta \ll \alpha$, and hence β is negligible, the experimental results of Figs. 2 and 3 are reduced to a single calibration for both flow regimes using the value of α deduced above. The assumption that β is negligible indicates that there is a negligible "apparent" conductivity variation in the inner region ($\eta < 12$) of a turbulent boundary layer, whereas there does exist an "apparent" viscosity variation in this region. Some physical justification for this can be made if the sublayer is viewed as essentially a laminar layer with velocity fluctuations within it produced by varying shear imposed by outer layers on the varying sublayer thickness. Thus, for gages that confine their thermal boundary layers to the region $\eta < 12$ over their effective lengths, Eq. (1) provides a basis for correcting turbulent data as shown in Fig. 3. A unique calibration for both flow regimes is not possible, but a criterion for agreement of results within specified tolerances can be developed. This criterion is

$$Lu_\tau/\sigma_\omega \nu_\omega < 0.51 \times 10^6 \times (\Delta\tau_\omega/\tau_\omega)_{max}{}^3 \quad (2)$$

which for $\Delta\tau_\omega/\tau_\omega = 4\%$ allowable error reduces to $Lu_\tau/\sigma_\omega \nu_\omega < 32$. This is a more restrictive criterion than Brown's[3] $Lu_\tau/\sigma_\omega \nu_\omega < 64$, and furthermore Brown expected complete agreement between laminar and turbulent calibrations. Brown's analysis is based, however, on an assumed linear turbulent velocity profile out to $\eta = 12$, and his experimental data was somewhat scattered ($\Delta\tau_\omega/\tau_\omega = \pm 15\%$), possibly partly transitional, and was obtained with a different gage having airgaps each side of the heated element.

Evaluation of $Lu_\tau/\sigma_\omega \nu_\omega$ for gages used by Pope,[5] for skin friction measurements in water gives a maximum value of 6. This indicates a level of agreement between laminar and turbulent calibrations that is well within 5%, which is confirmed experimentally by Fig. 4 showing a flat plate calibration in water.

It is to be noted that other experimental errors in measurement may also be present but the object of this analysis is to illustrate only inaccuracies caused by assuming the laminar calibration holds also in the turbulent regime.

References

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